Instability of gravitational solitons

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Felicity Eperon, HSR & Jorge Santos:
"Instability of supersymmetric microstate geometries"
arXiv:1607.06828
Joe Keir: "wave propagation on microstate geometries"
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Gravitational solitons do not exist

In 4d Einstein-Maxwell theory consider solutions which are

- Stationary (time-independent)
- Globally hyperbolic $\Rightarrow$ topology $\mathbb{R} \times \Sigma$
- Σ complete, asymptotically flat with a *single* asymptotic region

Such a solution could be regarded as a gravitational soliton

Theorem Serini, Einstein, Pauli, Lichnerowicz the only such solution is Minkowski spacetime!

This result extends to a large class of 4d supergravity theories

Breitenlöher, Maison & Gibbons 1988

Gravitational solitons do exist

But they require compact extra dimensions e.g.

- Static Kaluza-Klein bubble

$$ds^2 = -dt^2 + (1 - 2M/r)dz^2 + (1 - 2M/r)^{-1}dr^2 + r^2 d\Omega^2$$

$z \sim z + 8\pi M$

- Kaluza-Klein monopole

$$ds^2 = -dt^2 + ds^2(\mathrm{Euclidean\,Taub-NUT})$$

5d microstate geometries Bena & Warner 2005

Compact extra dimensions are not necessary - 5d supergravity admits a large class of solutions that are:

- Asymptotically flat
- Stationary: Killing vector field $\partial/\partial t$ timelike near infinity
- Globally hyperbolic
- Topology $\mathbb{R} \times \Sigma$ where Σ is complete and has non-trivial 2-cycles
- Supported by fluxes wrapping 2-cycles
- Rotating: non-zero angular momenta J_1, J_2
- No event horizon: these are not black holes

10d microstate geometries

Solitonic solutions of type IIB supergravity in 10 dimensions Maldacena & Maoz 2001, Lunin, Maldacena & Maoz 2002; Giusto, Mathur & Saxena 2004; Bena & Warner 2005, ...

Asymptotically $R^{1,4} \times S^1 \times T^4$

3 charges Q_1, Q_5, Q_P associated to wrapped D1 and D5-branes, Kaluza-Klein momentum around S^1

Some solutions contain arbitrary functions

"BPS" inequality $M \geq |Q_1| + |Q_5| + |Q_P|$ saturated when solution is *supersymmetric*

Fuzzball conjecture (Mathur)

Consider a black hole in 5 asymptotically flat dimensions

This has large entropy $S = A/(4G_5)$ so $N = e^S$ quantum microstates

Conjecture: some (all?) of these microstates are described geometrically by classical microstate geometries.

Much effort has gone into constructing explicit microstate geometries and counting them.

I will discuss the classical stability of these solutions.

Ergoregion instability

Non-supersymmetric microstate geometries have an ergoregion where $\partial/\partial t$ become spacelike Jejalla et al 2005

Spacetime with ergoregion but no horizon is likely to be unstable:

Friedman 1978, Moschidis 2016

- Linear perturbations can have negative energy in ergoregion
- Energy radiated to infinity always positive
- So negative energy can only get more negative (if there is no horizon)

Exponentially growing linearized perturbations exist: instability!

Cardoso, Dias, Hovdebo & Myers 2006

Supersymmetric microstate geometries

From now on: 5d *supersymmetric* 3-charge microstate geometry.

String theory folklore: (sufficient) supersymmetry $\Rightarrow$ stability.

I will argue that these solutions are actually unstable. The instability arises from a geometrical property of these solutions.

Evanescent ergosurface

Kiling field $V = \partial/\partial t$ timelike everywhere except on a timelike hypersurface $\mathcal{S}$. V is null on $\mathcal{S}$.

$\mathcal{S}$ is an ergosurface without an ergoregion Gibbons & Warner 2013

V is null on $\mathcal{S} \Rightarrow$ infinite redshift relative to infinity (supports fuzzball idea)

Geodesics

Define conserved energy for geodesic with momentum P: $E = -V \cdot P$. Then V causal $\Rightarrow E \geq 0$

$$V^b \nabla_b V_a = -V^b \nabla_a V_b = \nabla_a(-V^2/2)$$

RHS vanishes on $\mathcal{S}$ because $-V^2$ minimized there. Hence V is tangent to *null geodesics* on $\mathcal{S}$ (not true on a general ergosurface).

These geodesics have *zero energy*.

An evanescent ergosurface is a timelike hypersurface that is ruled by zero energy null geodesics.

Heuristic argument for instability

Consider a massive uncharged particle near $\mathcal{S}$ (e.g. KK mode of T^4, or a small black hole)

Leading order: particle moves on timelike geodesic, energy $E > 0$

Particle coupled to supergravity fields: slowly loses energy e.g. to gravitational radiation

E decreases: trajectory will approach energy-minimizing geodesic

Lowest energy geodesics are the zero energy *null* geodesics on $\mathcal{S}$

E is small but particle will have huge *local* energy so large backreaction, i.e, instability!

Nature of instability

Particle approaches orbit of V

Orbits of V are stationary relative to infinity: resist frame dragging caused by rotation of background spacetime so must have negative angular momentum (if background angular momentum is positive)

Particle will accelerate to approach a null orbit with (large) negative angular momentum. (Small) energy and (large) positive angular momentum will be radiated to infinity.

So instability involves large backreaction near $\mathcal{S}$ with a small change in energy, and large reduction in angular momentum of background.

Guess: collapse to form an almost supersymmetric black hole - BMPV or black ring.

Instability in supergravity

Can we see an instability involving only massless supergravity fields?

Heuristic argument involves coupling to radiation so instability likely to be *nonlinear*

But even *linear* stability is not obvious...

Linear stability

Can find decoupled linear perturbations behaving as massless scalar

Cardoso, Dias & Myers 2007

Supersymmetry $\Rightarrow$ energy always non-negative. If energy small initially then energy remains small.

For a supersymmetric microstate geometry, this argument excludes *exponential* growth.

However: the conserved energy degenerates on $\mathcal{S}$. For example energy-momentum current for massless scalar:

$$j^a = -T^a{}_b V^b = -\partial^a \Phi V \cdot \partial \Phi + \frac{1}{2} V^a (\partial \Phi)^2$$

vanishes on $\mathcal{S}$ if $\partial_a \Phi \propto V_a$ there.

Small conserved energy does not prevent $\partial \Phi$ becoming large on $\mathcal{S}$.

Nonlinear stability

Let's *assume* that we do have linear stability.

Proofs of nonlinear stability (e.g. Minkowski spacetime) rely on sufficiently rapid decay of linear perturbations to infinity or across a black hole horizon

For example: AdS has no decay, suggests nonlinear instability Dafermos & Holzegel 2006, confirmed numerically Bizon & Rostworowski 2011.

Minkowski or asymptotically flat black holes: t^{-p} decay, expect nonlinear stability.

So how fast do linear perturbations decay in supersymmetric microstate geometries?

Quasinormal modes

We can investigate decay of waves in these geometries by finding quasinormal modes:

$$\Phi(t,x) = e^{-i\omega t}F(x) \qquad \omega = \omega_R + i\omega_I \qquad \omega_I < 0$$

Boundary conditions: outgoing at infinity.

For black holes, it is known that there is a connection between quasinormal modes and null geodesics via geometric optics Ferrari & Mashoon 1984.

Quasinormal modes correspond to *trapped* null geodesics of the black hole.

Trapping

A null geodesic on the photon sphere $r = 3M$ of a Schwarzschild black hole is *trapped*: it remains forever in a finite region of space.

Kerr has trapped null geodesics for some range $r_1 < r < r_2$.

This trapping is *unstable*: if perturbed, such a geodesic will escape to infinity or fall into the black hole.

Geometric optics/WKB lets us determine ω from properties of these geodesics. For example in Kerr Yang et al 2012

$$\Phi = e^{-i\omega t} e^{im\phi} \Theta_\ell(\theta) R(r) \qquad |m| \leq \ell$$

For $\ell \gg 1$ we have $\omega_R/m \approx E/L$ where L is angular momentum of geodesic. $\omega_I = \mathcal{O}(1)$ is determined by the timescale for the instability of the trapping.

Stable trapping in microstate geometries

The zero energy null geodesics on $\mathcal{S}$ exhibit *stable* trapping because they minimize the energy.

So geometric optics suggests there are quasinormal modes with very small ω_I, i.e., very slow decay. This is because the waves have to tunnel (classically) through a large potential barrier to escape.

Since these geodesics have $E = 0$ we expect the associated quasinormal modes to have $\omega_R \approx 0$.

Calculation of quasinormal modes

We have calculated ω for the most symmetrical 2-charge and 3-charge microstate geometries Maldacena & Maoz 2001, Giusto, Mathur & Saxena 2004

These geometries have 2 rotational symmetries $\partial/\partial\phi$, $\partial/\partial\psi$ and a "hidden" symmetry that enables separation of variables:

$$\Phi = e^{-i\omega t} e^{im_\phi \phi} e^{im_\psi \psi} \Theta_\ell(\theta) R(r) \qquad \ell \geq |m_\phi| + |m_\psi|$$

For $m_\phi < 0$ and $\ell \gg 1$ a matched asymptotic expansion gives quasinormal modes with

$$\omega_R = \mathcal{O}(1) \qquad \omega_I = -\beta e^{-2\ell \log \ell}$$

Very slow decay at large ℓ!

More general microstate geometries

These calculations were for the most symmetrical microstate geometries.

The slowly decaying quasinormal modes are localized around the zero energy null geodesics on $\mathcal{S}$.

These null geodesics exist whenever there is an evanescent ergosurface so expect slowly decaying quasinormal modes for any supersymmetric microstate geometry..

Slow decay

Proofs of nonlinear stability require *uniform* decay of some *non-degenerate* energy functional $E_1(t)$ quadratic in $\partial\Phi$

For example: could take $E_1(t)$ to be energy according to "zero angular momentum" observers with velocity parallel to $-dt$

Ideally $E_1(t) \leq g(t)E_1(0)$ with $g(t)$ independent of Φ and $g(t) \to 0$ as $t \to \infty$

Not possible because of trapping Sbierski 2013 but maybe $E_1(t) \leq g(t)E_2(0)$ where E_2 quadratic in $\partial\Phi$ and $\partial^2\Phi$

Slow decay

$$E_1(t) \leq g(t) E_2(0)$$

Black holes (unstable trapping): $g(t) = t^{-p}$ Dafermos & Rodnianski

Previous examples with stable trapping: AdS black holes Holzegel & Smulevici 2013, ultracompact neutron stars Keir 2014:

$$g(t) = (\log(2+t))^{-2}$$

Waves decay at least this fast in a large class of asymptotically flat spacetimes that are either *strictly* stationary or contain an event horizon Moschidis 2015

Our quasinormal modes imply that the decay must be *even slower* than this for supersymmetric microstate geometries. Slowest decay of any known asymptotically flat spacetime!

Comparison with black holes

Extremal Reissner-Nordstrom: t^{-p} decay, slowest decaying modes have low ℓ Aretakis 2012

Supersymmetric microstate geometry: decay slower than $(\log(2+t))^{-2}$, slowest decay modes have large ℓ

Qualitative differences between behaviour of linear fields in black hole geometries and microstate geometries

Nature of nonlinear instability

Slowest decay for high ℓ modes localized around $\mathcal{S}$: suggests nonlinear instability will be short-distance effect

Maybe stable trapping leads to formation of tiny black hole as in AdS instability

This would behave as in our heuristic argument, accelerates to high speed so larger backreaction

Natural guess for endpoint: collapse to near-supersymmetric black hole

Remark: "black lens" solutions Kunduri & Lucietti 2014 also have evanescent ergosurface so likely to suffer similar instability...

Making it rigorous (Keir)

For the most symmetrical microstate geometries:

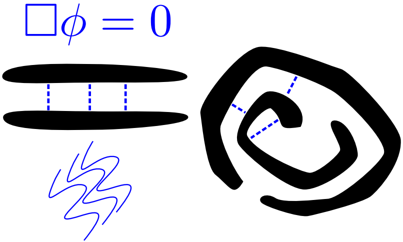
- Solutions of wave equation are bounded
- Uniform decay $E_1(t) \leq g(t)E_2(0)$ cannot be faster than

$$g(t) = \left[\frac{\log\log(2+t)}{\log(2+t)}\right]^2$$

Obstacle problem

Logarithmic decay outside of an arbitrary obstacle (Burq, 1998)

$$\mathcal{E}^{(N)}_{\text{local}}(t) \lesssim \frac{1}{(\log(2+t))^2}\mathcal{E}^{(N)}_{(2)}(0)$$



Waves in supersymmetric microstate geometry decay slower than for any arrangement of mirrors in flat spacetime!

Growth of energy

Keir will prove (in forthcoming work) that there are solutions with small $E_1(0)$ for which $E_1(t)$ can become arbitrarily large.

Such solutions have large $E_2(0)$. If $E_2(0)$ small then looks like $E_2(t)$ can become large etc.

If so, whatever (Sobolev) norm one imposes on the initial data, this norm can become large in time evolution.

This looks like an instability even for *linear* perturbations.

These results hold for a large class of microstate geometries.

Alternative endpoint Marolf, Michel, Puhm 2016

Instability is a short-distance effect: stringy α' corrections may be important

An entropic argument applied to stringy microstates suggests that these effects will lead to a different endpoint: microstate geometry with α'-scale features

"A rough end for smooth microstate geometries": non-generic smooth microstate geometries evolve to generic rough microstate geometries

Summary

Supersymmetric microstate geometries are conjectured to describe individual black hole microstates.

Heuristic ”particle” argument indicates nonlinear instability.

Stable trapping of null geodesics implies very slow decay of linear perturbations: slower than any known asymptotically flat spacetime.

This decay is qualitatively different from decay in a black hole geometry.

Decay too slow for proving nonlinear stability so expect instability.

Possible endpoints: collapse to near-supersymmetric black hole, or stringy corrections become important.