Negative Energy and Quantum Stress Tensor Fluctuations

King's College, London
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Larry Ford
Tufts University

Early work on quantum field theory in curved spacetime

Particle creation in an expanding universe

Schrödinger (1939) “alarming phenomena”

Parker (1969)

Particle creation by black holes

Hawking (1974)

Work at King's on quantum fields in curved space in the 1970's

Definition of the expectation value of a quantum stress tensor operator

Application to understanding back reaction in the Hawking effect

Violation of the classical energy conditions: negative energy density

QFT in deSitter - Bunch-Davies vacuum

Expectation value of a quantum stress tensor operator

Needed for the semiclassical theory of gravity:

$$G_{\mu\nu} = 8\pi \langle T_{\mu\nu} \rangle$$

Subtle aspects of $\langle T_{\mu\nu} \rangle$:

Formally infinite - needs regularization and renormalization

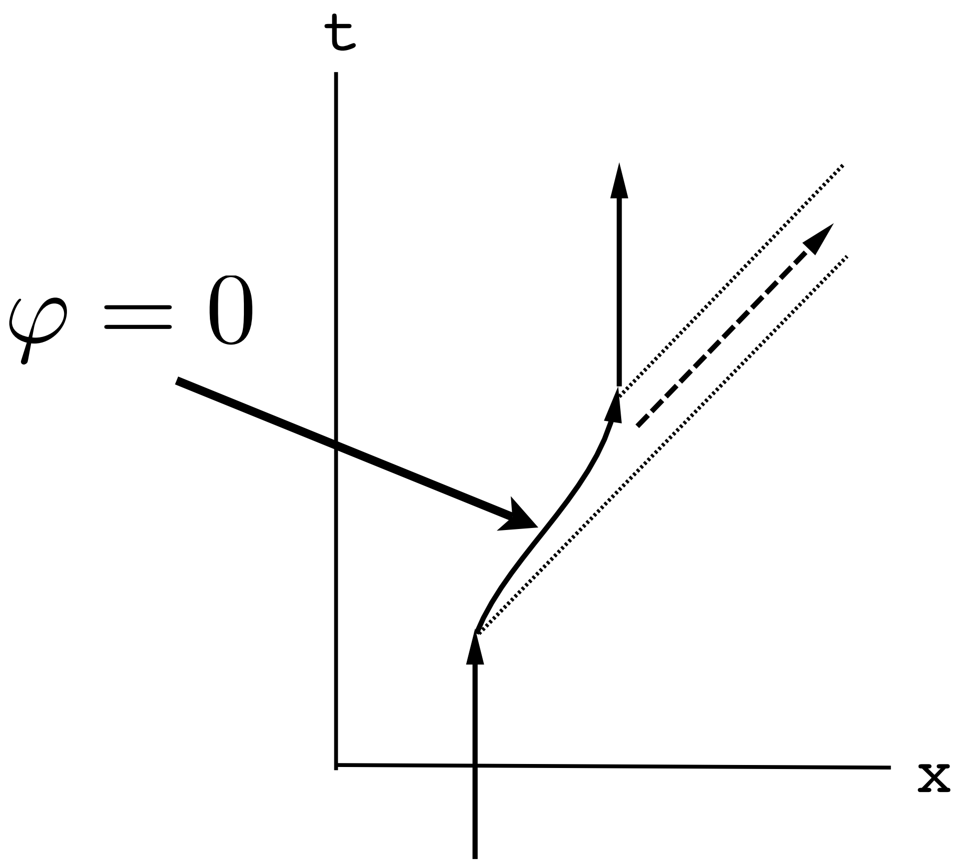
Conformal anomaly - $\langle T^{\mu}_{\mu} \rangle \neq 0$ even if $T^{\mu}_{\mu} = 0$ classically.

Deser, Duff & Isham 1976

Violation of classical energy conditions

Particle creation by moving mirrors in 2D spacetime

Fulling & Davies 1976



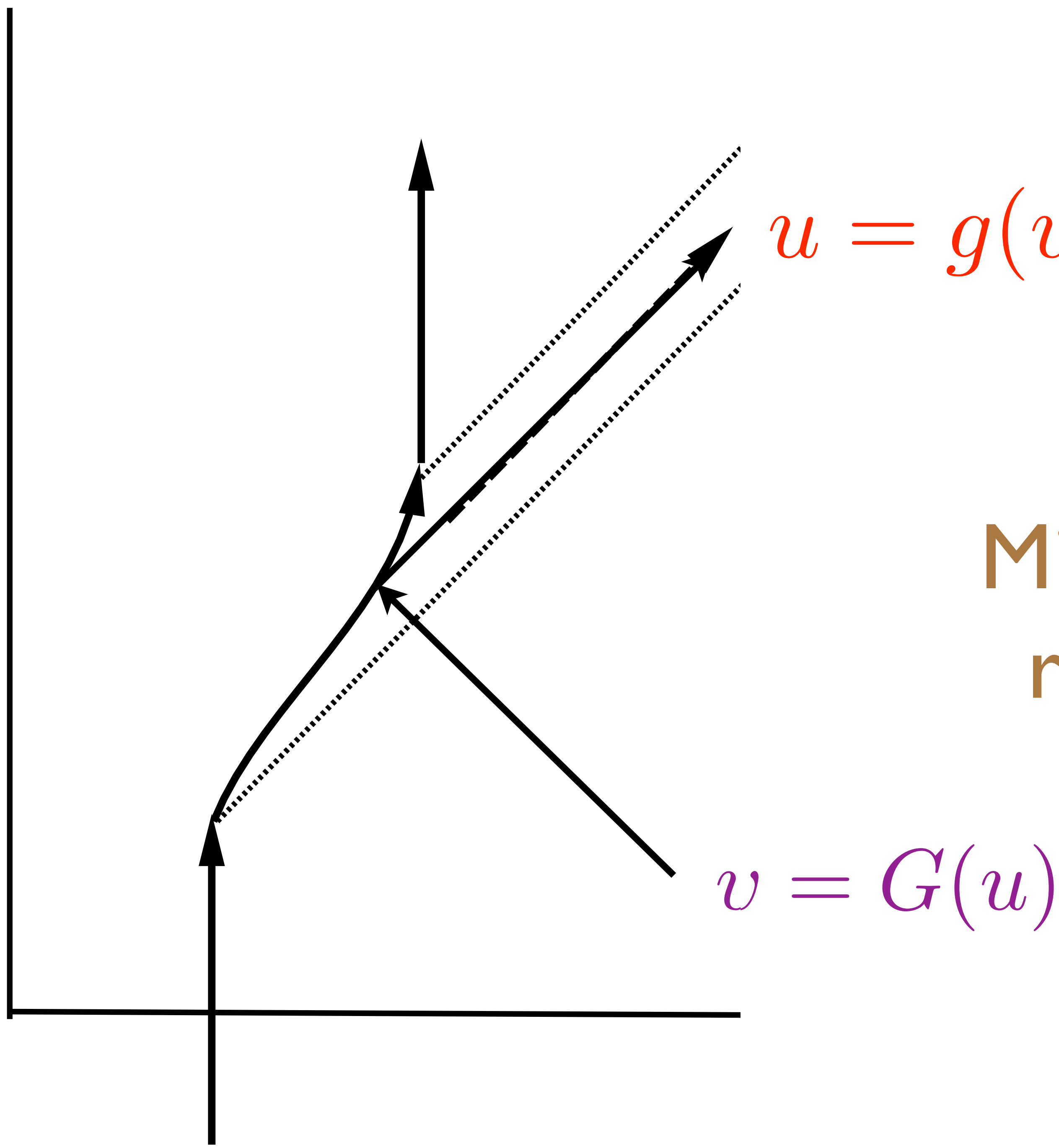
Assume that a massless scalar field vanishes at the mirror.

Null coordinates:

$u = t - x$ constant on right-moving rays

$v = t + x$ constant on left-moving rays

Quantum radiation is emitted to both sides of the mirror, but we consider only the right side.



Mirror maps incoming null rays into outgoing ones.

Superposition of (+) and (-) frequency parts

Mode function which vanishes on the mirror:

$$f_k(x) = \frac{1}{\sqrt{4\pi\omega}} \left(\underbrace{e^{-i\omega v}}_{\text{incoming}} - \underbrace{e^{-i\omega G(u)}}_{\text{outgoing}} \right)$$

Particle creation!

Energy flux radiated by the mirror:

$$F(u) = \langle T^{xt} \rangle = \frac{1}{48\pi}\left[3\left(\frac{G''}{G'}\right)^2 - 2\left(\frac{G'''}{G'}\right)\right]$$

In terms of the mirror's velocity:

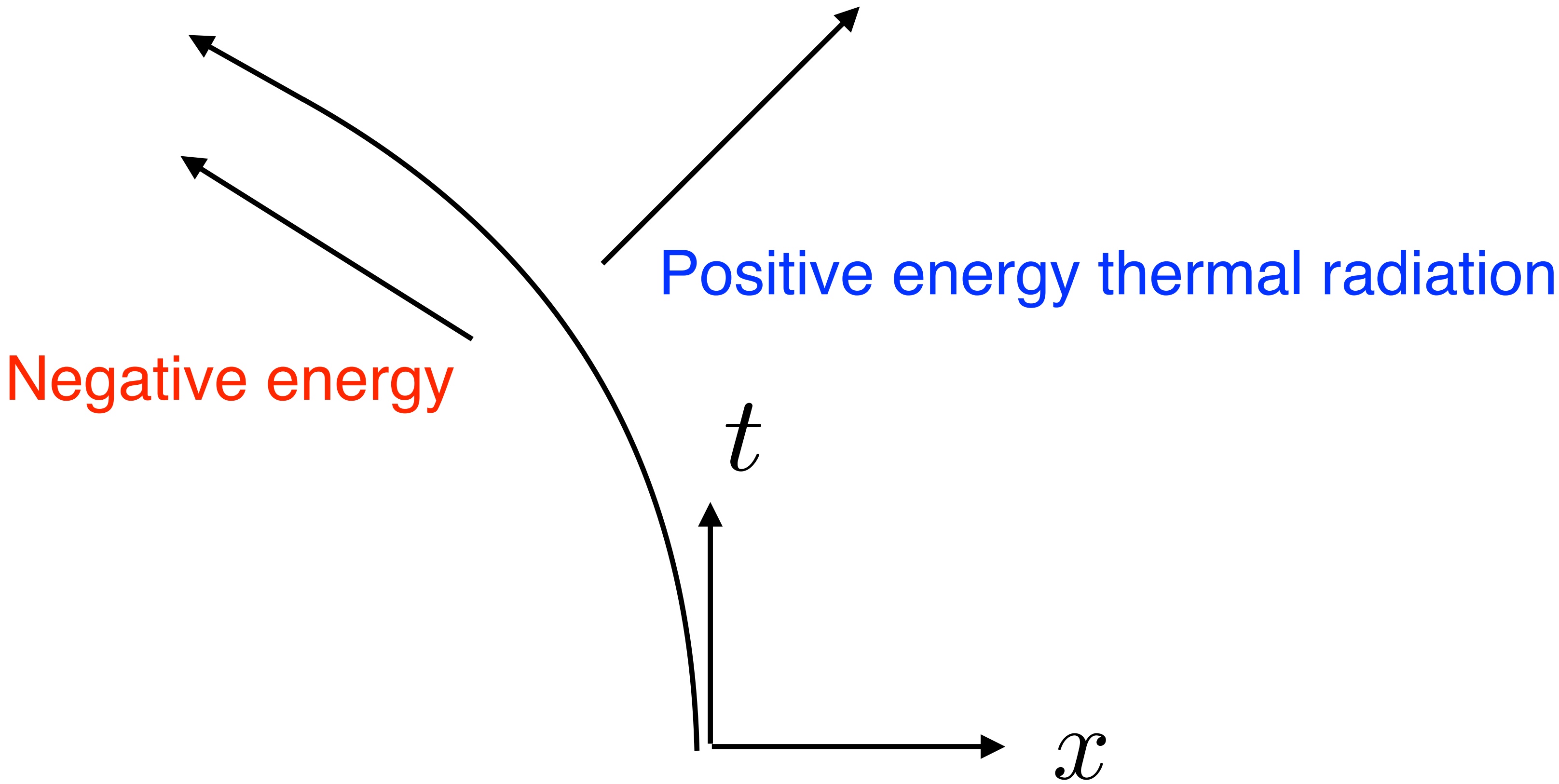
$$F = -\frac{(1-v^2)^{1/2}}{12\pi(1-v)^2}\,\frac{d}{dt}\left[\frac{\dot{v}}{(1-v^2)^{3/2}}\right]$$

Non-relativistic limit:

$$F \approx -\frac{\ddot{v}}{12\pi}$$

Flux can be either positive or negative (an example of negative energy in quantum field theory).

A mirror trajectory which models the Hawking effect.

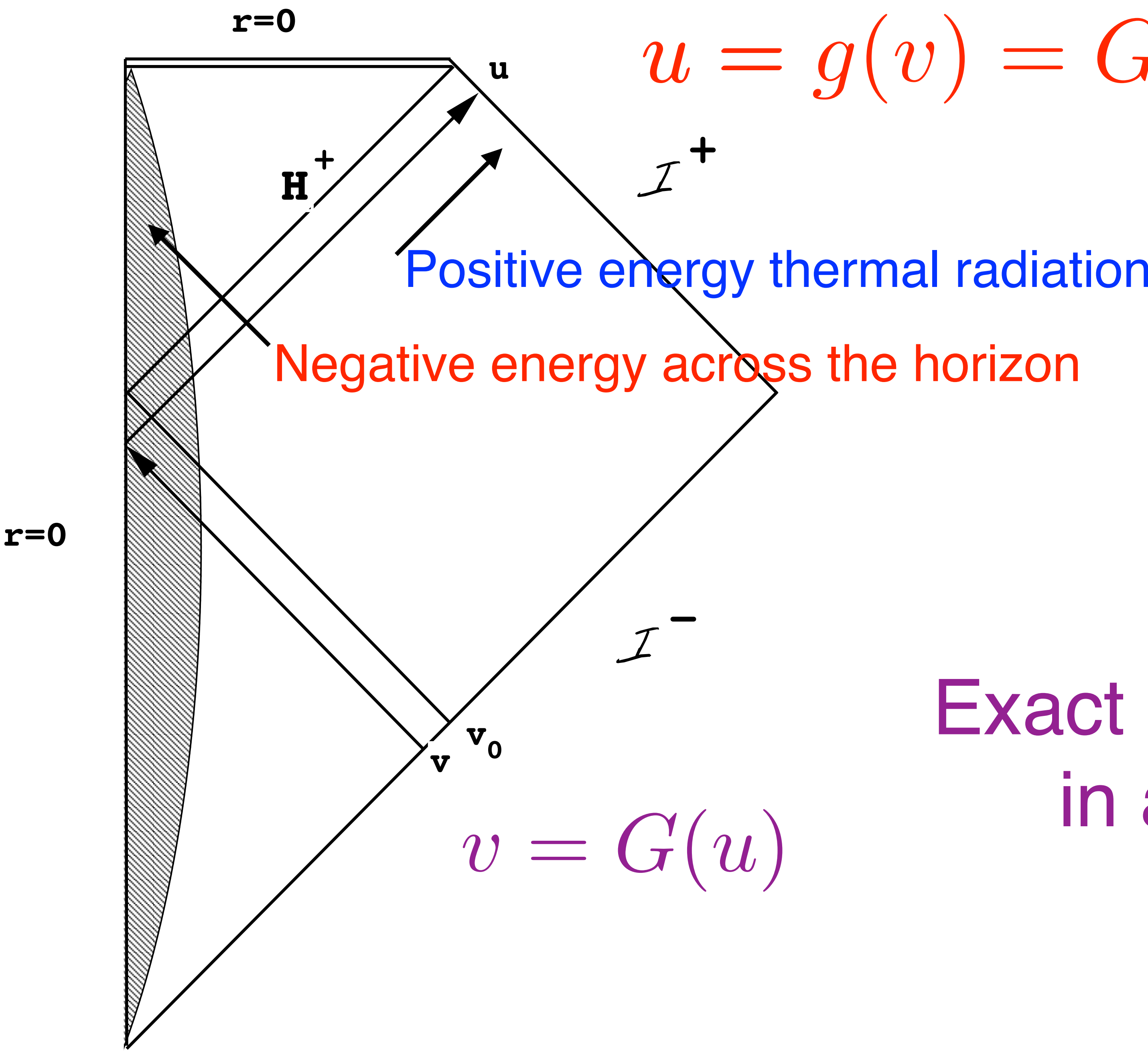


$$T = \frac{\kappa}{2\pi k_B}$$

$$x(t) = -t + A\, e^{-2\kappa t}$$

Evaporating Black Hole Spacetime

$$u = g(v) = G^{-1}(v)$$



$$v = G(u)$$

Exact solution for the stress tensor in a two dimensional model

Davies, Fulling & Unruh 1976

Possible effects of negative energy:

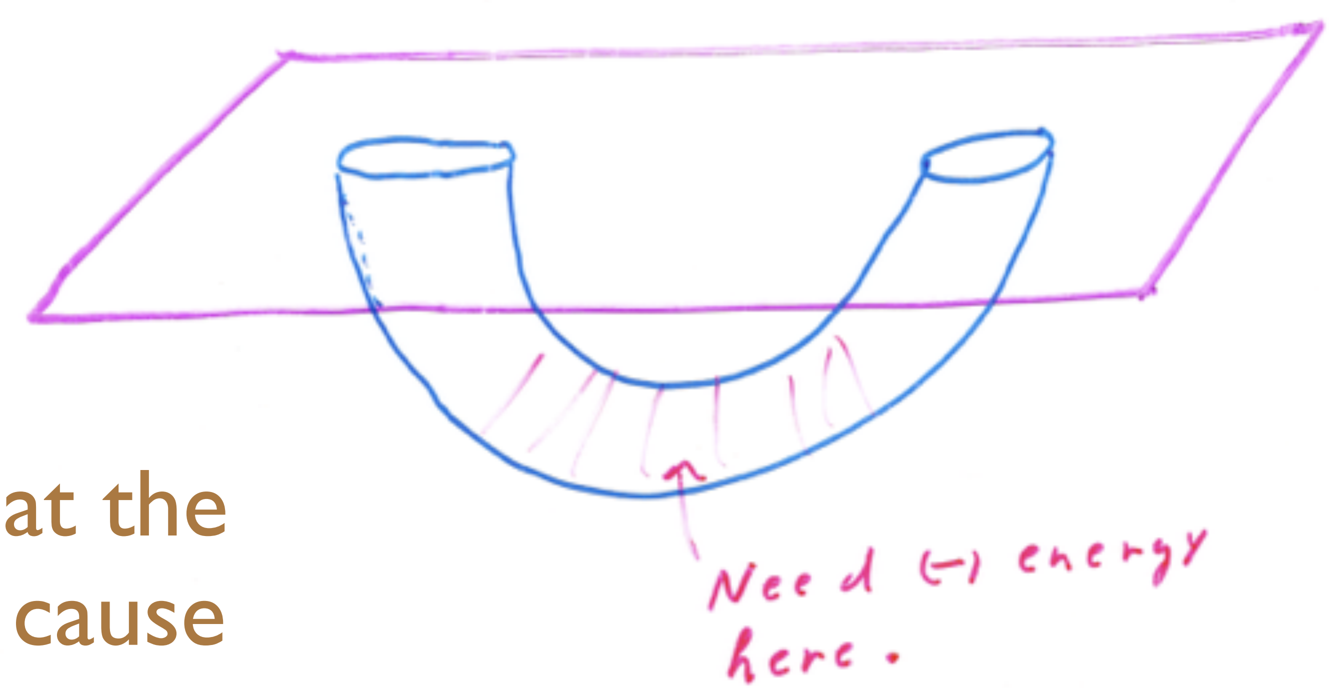
Repulsive gravity

Singularity avoidance

Violation of the weak energy conditions allows the singularity theorems to be evaded.

Traversable wormholes

Morris & Thorne



Negative energy is needed at the throat of the wormhole to cause light rays to defocus.

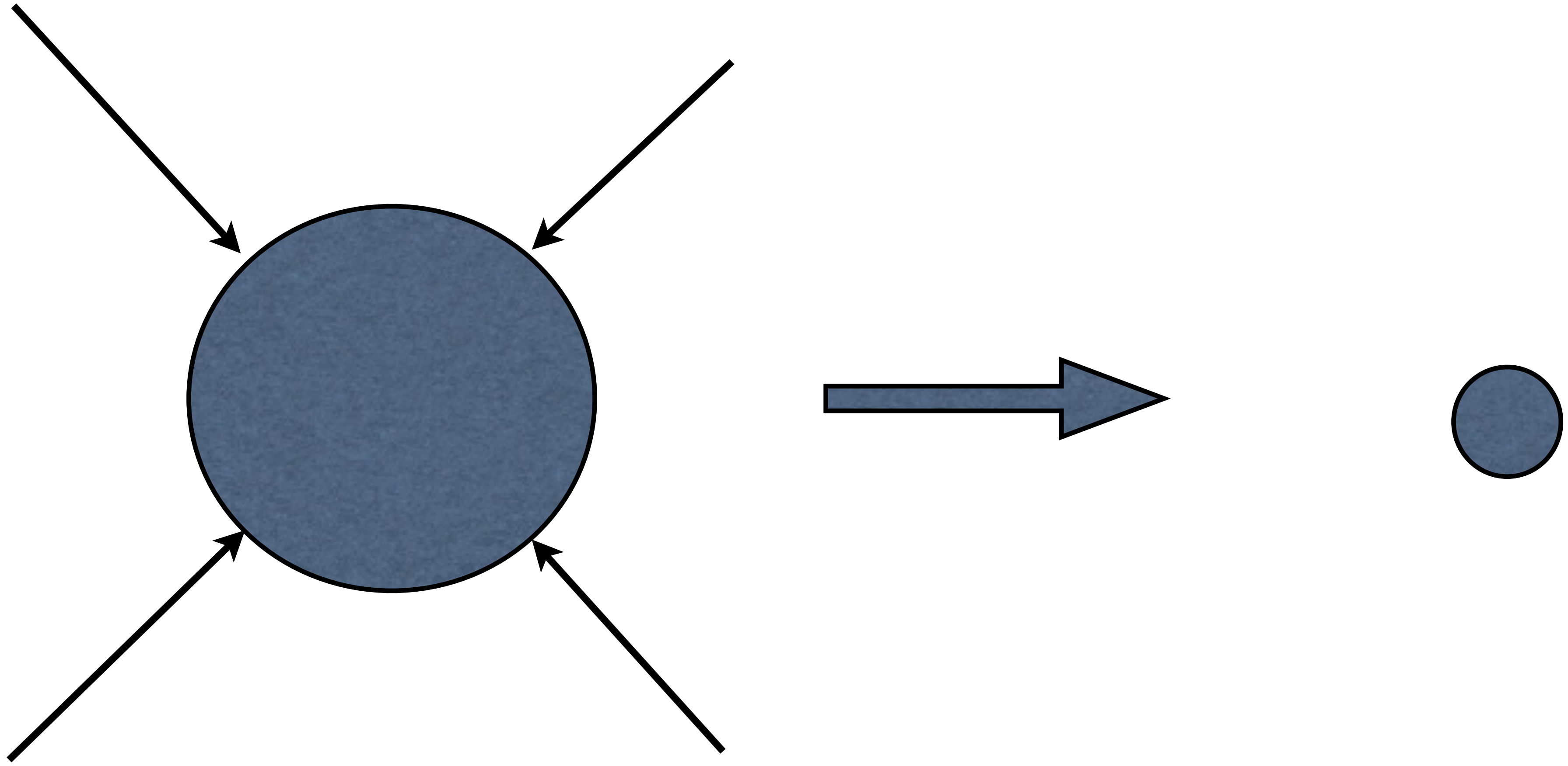
Faster than light travel - Alcubierre warp drive

Time Machines

Can modify a traversable wormhole or warp drive to travel backwards in time

Hawking's Theorem: Negative energy is essential to build a time machine.

Violations of the second law of thermodynamics



Shine negative energy on a black hole and reduce its horizon area.

Limits on the effects of negative energy

Quantum inequalities -

$$\int \langle T_{\mu\nu} \rangle \, u^\mu \, u^\nu \, g(\tau, \tau_0) \, d\tau \geq -\frac{C}{\tau_0^d}$$

$g(\tau, \tau_0)$ = sampling function

C = positive constant

τ_0 = sampling time

d = spacetime dimension

LF 1978, subsequent work by Roman, Fewster, Eveson, Flanagan, Teo, and others

Results for general sampling functions:

$$\rho = \langle T_{\mu\nu} \rangle u^\mu u^\nu$$

Two dimensions (1+1)

$$\int_{-\infty}^{\infty} \rho(\tau)\, g(\tau)\, d\tau \geq -\frac{1}{24\pi} \frac{[g'(\tau)]^2}{g(\tau)}$$

(Flanagan) Optimum bound

Four dimensions (3+1)

$$\int_{-\infty}^{\infty} \rho(\tau)\, g(\tau)\, d\tau \geq -\frac{1}{16\pi^2} [(\sqrt{g(\tau)})'']^2$$

(Fewster&Eveson)

Quantum stress tensor fluctuations

$\langle T_{\mu\nu} \rangle = 0$ in the Minkowski vacuum

but the vacuum is not an eigenstate of

$\Longrightarrow$ energy density fluctuations, including negative fluctuations

Quantum stress tensor fluctuations lead to (passive) spacetime geometry fluctuations.

Probability distribution for quantum stress tensor fluctuations

Need to average the operator over a finite spacetime region

Must be a skewed, non-Gaussian distribution

In general, the odd moments are non-zero.

Expect the probability distribution to have a lower cutoff at the quantum inequality bound on expectation values

A massless scalar field in two dimensions:

C. Fewster, T. Roman & LF, (2010)

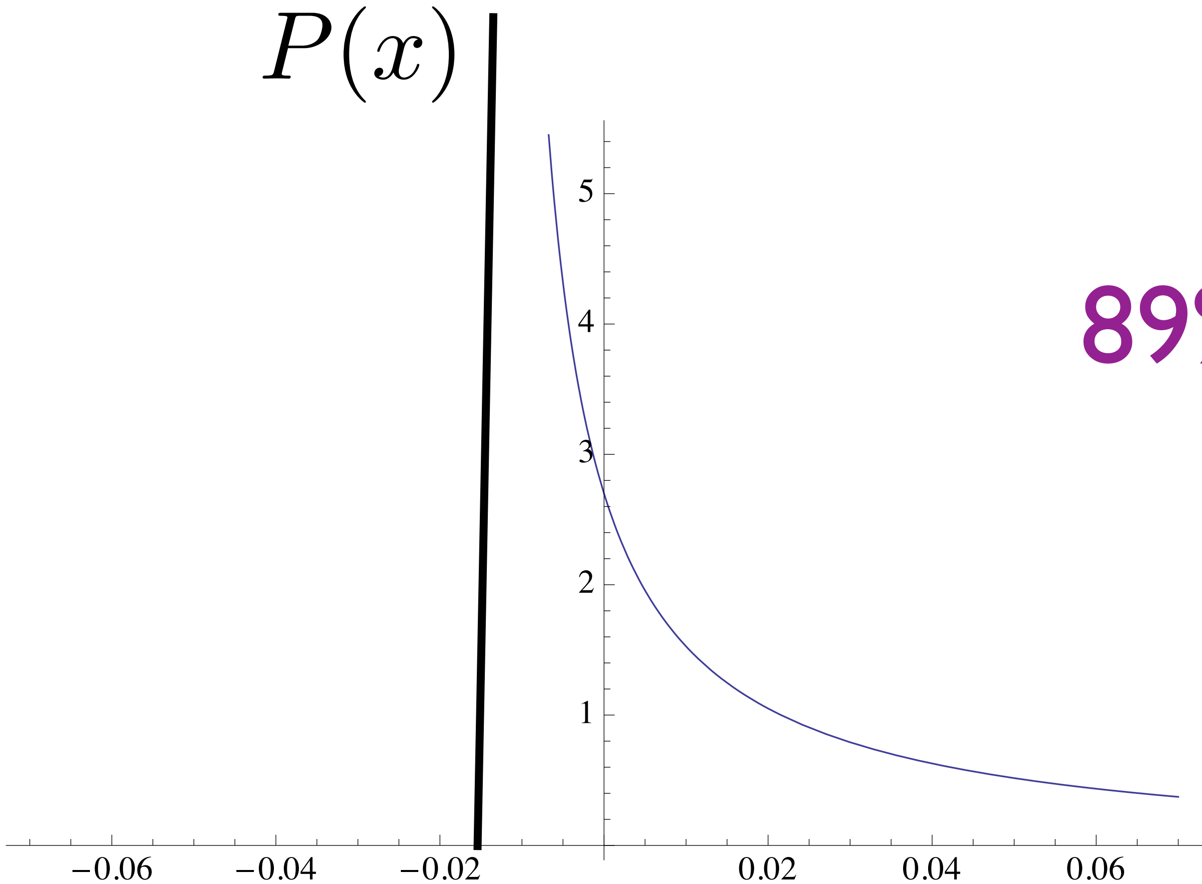
$$P(x) = \frac{\pi^{1/24}}{\Gamma(1/24)} \left(x + \frac{1}{24\pi} \right)^{-23/24} e^{-\pi(x+1/24\pi)}$$

$$P(x) = 0 \quad x < -\frac{1}{24\pi}$$

$$x = u\,\tau^2$$

$$u = \frac{1}{\sqrt{\pi}\tau} \int_{-\infty}^{\infty} T_{tt}(x,t)\, e^{-t^2/\tau^2}\, dt$$

$P(x)$



89% chance of finding u < 0

$$x = \tau^2\, u$$

Lorentzian sampled EM energy density in four dimensions

C. Fewster, T. Roman & LF (2012)

$$x = (4\pi\,\tau^2)^2 \int_{-\infty}^{\infty} :T_{tt}(\mathbf{x},t):\; g(t,\tau)\,dt \qquad g(t,\tau) = \frac{\tau}{\pi(t^2+\tau^2)}$$

Asymptotic form of the probability distribution:

$$P(x) \sim c_0\, x^{-2}\, \mathrm{e}^{-a\,x^{1/3}}$$

$$x \gg 1 \qquad c_0 \approx a \approx 0.96$$

Large positive fluctuations are more likely than one might expect, and eventually vacuum fluctuations dominate over thermal fluctuations.

Effects of a stress tensor measured in a finite time interval

C. Fewster & LF 2015

Let the sampling function $f(t)$ have compact support, so $f(t) = 0$ outside of a finite time interval.

Its Fourier transform $\hat{f}(\omega)$ will fall more slowly than an exponential, leading to a larger probability for large fluctuations.

Consider $\hat{f}(\omega) = \mathrm{e}^{-|\omega|^{\alpha}}$

$\alpha = 1$ Lorentzian

$\alpha < 1$ a class of compactly supported sampling functions

Asymptotic form for the probability distribution:

$$P(x) \sim c_0\, x^b\, \mathrm{e}^{-a x^c}$$

where $c = \dfrac{\alpha}{3}$

Implications:

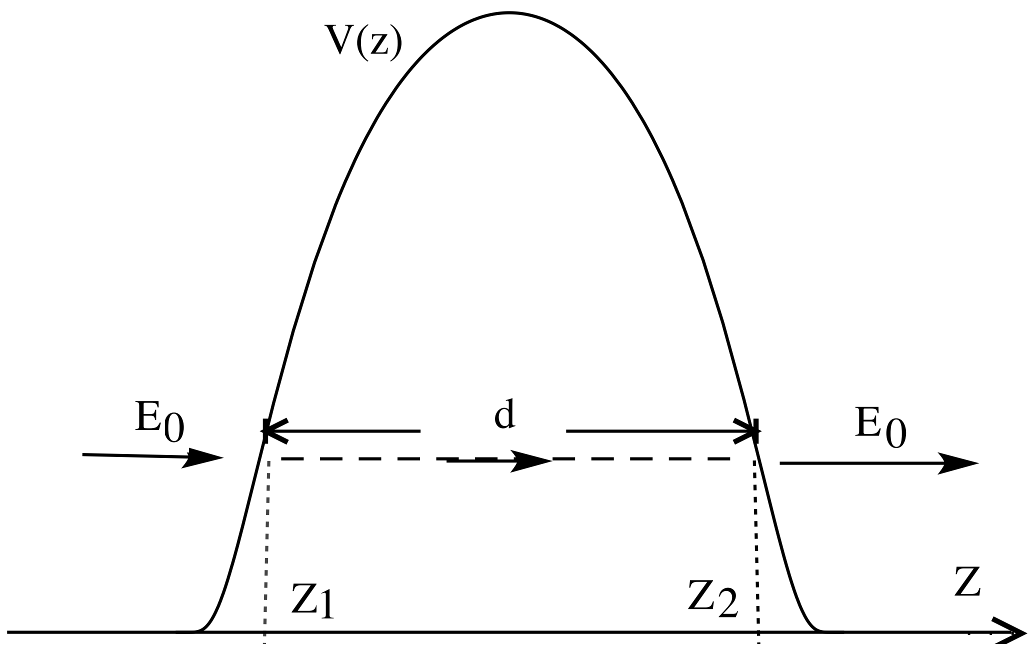
1) The form of the switching function is very important.

2) Switching in a finite time interval can produce large stress tensor fluctuations.

An application of large stress tensor fluctuations to quantum barrier penetration

H. Huang and LF (2016)

Quantum tunneling probability in the WKB approximation:



$$P_{\mathrm{WKB}} = \mathrm{e}^{-G}$$

$$G = 2 \int_{z_1}^{z_2} \sqrt{2\, m\, [V(z) - E_0]}\, dz$$

$$= 2 \sqrt{2\, m\, [V(z_m) - E_0]}\; d$$

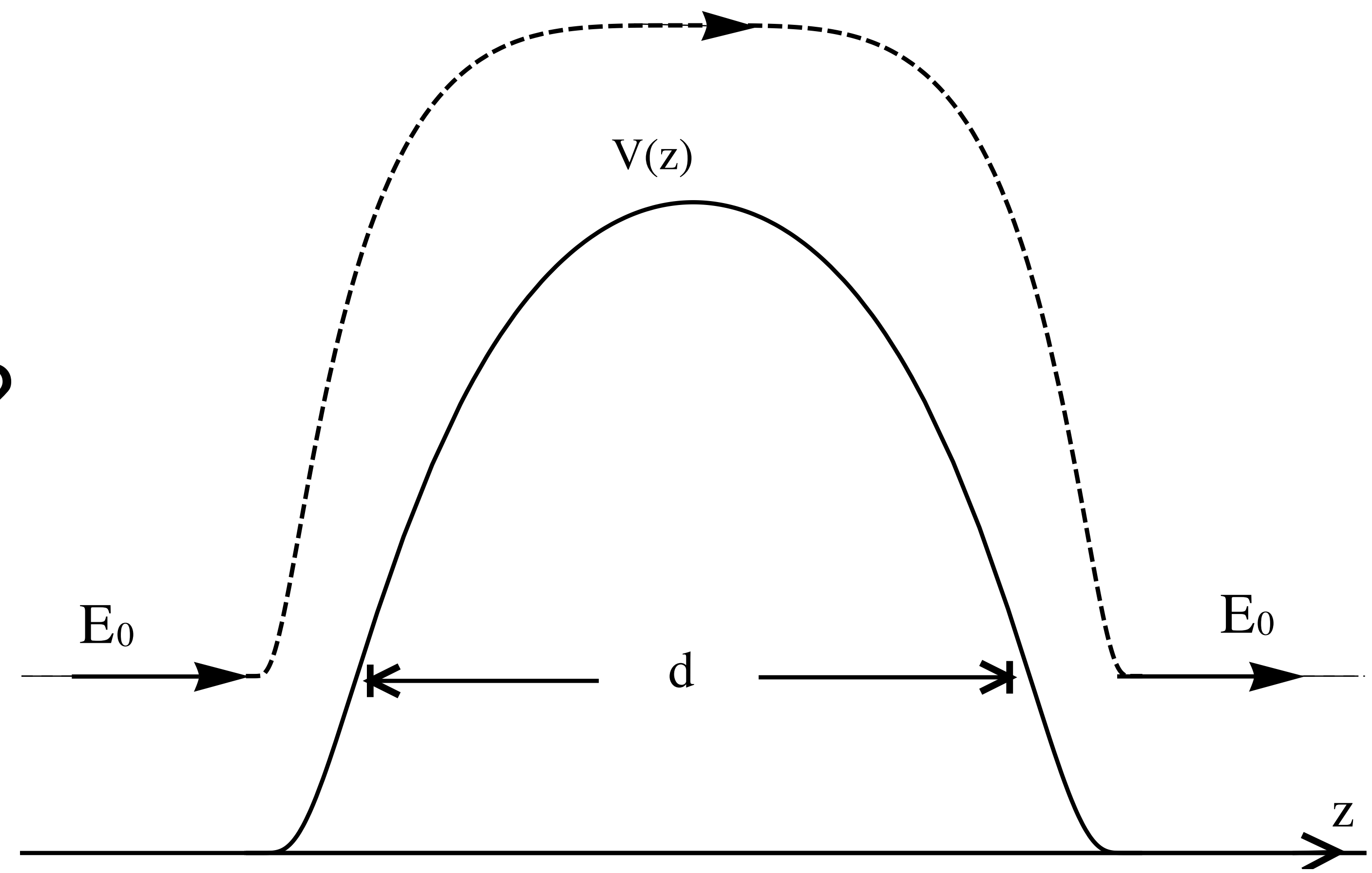
Let $v_1 = \sqrt{2\,[V(z_m) - E_0]/m}$ $\qquad z_1 \leq z_m \leq z_2$

Then $$G = 2\, v_1 \left(\frac{d}{\lambda_C} \right)$$

Compton wavelength

Effects of large radiation pressure fluctuations

Can a large vacuum fluctuation push the particle over the barrier?



Analog of thermal activation at finite temperature.

Define the cumulative probability, the probability for a fluctuation over a given threshold, by

$$P_>(x) = \int_x^\infty P(y)\, dy$$

$$\approx \frac{c_0}{a\, c}\, x^{1+b-c}\, \mathrm{e}^{-a x^c} = \mathrm{e}^{-F(x)} \qquad c = \frac{\alpha}{3}$$

$$F(x) = a\, x^c - (1 + b - c)\, \ln x - \ln\left(\frac{c_0}{ac}\right)$$

Vacuum fluctuations will dominate over quantum tunneling if $F(x) < G$

The form of the sampling function should be determined by the physical system. In this case, perhaps the shape of the potential and of the particle wave packet.

Radiation pressure fluctuations might be able to explain the observed cross sections for the fusion of heavy nuclei, such as $O^{16} + Sm^{144}$

Some implications of large positive EM energy density fluctuations:

1) Black Hole nucleation in flat spacetime

Rate estimates (based on the Lorentzian result):

$$m = 1000\, m_p$$

1 per present horizon volume per age of Universe

$$m = 400\, m_p$$

1 per cubic centimeter per second

Must be surrounded by negative energy and disappear quickly

2) Boltzmann brain nucleation

Assume M = 1kg, size = 10 cm, duration = 0.3 s

Rate: $e^{-10^{26}}$

Compare to Page's estimate of $e^{-10^{50}}$

The much larger rate might complicate attempts at anthropic reasoning.

“Boltzmann Brains”

3) Implications for the short distance structure of spacetime

Carlip, Mosna & Pitelli
(model based in the 2D probability distribution)

The large positive fluctuations tend to lend to strong focussing on small scales, somewhat above the Planck length. This leads to "asymptotic silence" where spacetime breaks into small causally disconnected regions.

4) Possible non-Gaussian contribution to primordial density or gravity wave perturbations

5) Other large passive fluctuations of the gravitational field driven by stress tensor fluctuations

Summary

1) Work at King's in the 1970's was crucial in the development of quantum field theory in curved spacetime.

2) This included the conformal anomaly, calculations of $\langle T_{\mu\nu} \rangle$, and back reaction of Hawking radiation.

3) Also studies of negative energy effects, and their quantum inequality limits.

4) Probability distributions for stress tensor fluctuations - lower bound is the quantum inequality limit.

5) The probability of large positive fluctuations can be rather large and may have interesting physical consequences.